AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TYM TEST 16 UNISION SOLUTION Date 08-12-2019

[PHYSICS]

1. The effective acceleration of a bob in water $= g' = g\left[1 - \frac{\sigma}{\rho}\right]$

where σ and ρ are the densities of water and the bob respectively. Since, the periods of oscillation of the bob in air and water are given as

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ and } T' = 2\pi\sqrt{\frac{l}{g'}}$$

$$\therefore \quad \frac{T}{T'} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{g\left(1 - \frac{\sigma}{\rho}\right)}{g}}$$

$$= \sqrt{1 - \frac{\sigma}{\rho}} = \sqrt{1 - \frac{1}{\rho}} \qquad [\because \sigma = 1]$$

Putting $\frac{T}{T'} = \frac{1}{\sqrt{2}}$

We obtain, $\frac{1}{2} = 1 - \frac{1}{\rho} \Rightarrow \rho = 2$

2. When the elevator is at rest, its time period is given by

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{10}}$$

When the elevator accelerates upwards, its time period becomes

$$T' = 2\pi\sqrt{\frac{l}{g + a}} = 2\pi\sqrt{\frac{l}{10 + 10}}$$

$$= 2\pi\sqrt{\frac{l}{20}}$$

$$= 2\pi\sqrt{\frac{l}{10}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{T}{\sqrt{2}}$$

3. Velocity of bob $v = \sqrt{2gl(1 - \cos\theta)}$

$$= \sqrt{2 \times 9.8 \times 2 \times (1 - \cos 60^\circ)}$$

$$v = \sqrt{2 \times 9.8} \text{ m/s}$$

4.
5.

6. Let displacement equation of particle executing SHM is

$$y = a \sin\omega t$$

As particle travels half of the amplitude from the equilibrium position, so

$$y = \frac{a}{2}$$

Therefore, $\frac{a}{2} = a \sin\omega t$

or $\sin\omega t = \frac{1}{2} = \sin\frac{\pi}{6}$

or $\omega t = \frac{\pi}{6}$

or $t = \frac{\pi}{6\omega}$ or

or $t = \frac{\pi}{6\left(\frac{2\pi}{T}\right)}$ $\left(\text{as } \omega = \frac{2\pi}{T}\right)$ 60. Given,

and

or $t = \frac{T}{12}$

Hence, the particle travels half of the amplitude from equilibrium in $\frac{T}{12}$ s.

7.
8.
9.
10.

11. Time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

It is independant of the mass of the bob. Therefore time period of the pendulum will remain T.

12.

13. Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

But at temperature $\theta°$ C, increase in length of pendulum,

$$\frac{\Delta l}{l} = \alpha\Delta\theta$$

$\therefore$ $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$

or $\frac{\Delta T}{T} = \frac{1}{2} \times 9 \times 10^{-7} \times (30 - 20)$

$$= \frac{1}{2} \times 9 \times 10^{-7} \times 10$$

$$= 4.5 \times 10^{-6}$$

or $\Delta T = 4.5 \times 10^{-6} \times 0.5$

$= 2.25 \times 10^{-6}$ s

14.
15.

16. **(b)** Acceleration of simple harmonic motion is

$$a_{max} = -\omega^2 A$$

or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{\omega_1^2}{\omega_2^2}$ (as A remains the same)

or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{(100)^2}{(1000)^2} = \left(\frac{1}{10}\right)^2 = 1:10^2$

17. **(d)** $v = \frac{dy}{dt} = A\omega \cos \omega t = A\omega\sqrt{1-\sin^2 \omega t}$

$= \omega\sqrt{A^2 - y^2}$

Here, $y = \frac{a}{2}$

$\therefore \quad v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\sqrt{\frac{3a^2}{4}} = \frac{2\pi}{T}\frac{a\sqrt{3}}{2} = \frac{\pi a\sqrt{3}}{T}$

18. **(b)** Acceleration $\propto$ – (displacement).

$A \propto -y$

$A = -\omega^2 y$

$A = -\frac{k}{m}y$

$A = -ky$

Here, $y = x + a$

$\therefore$ acceleration $= -k(x + a)$

19. **(b)** Use the law of conservation of energy. Let x be the extension in the spring.

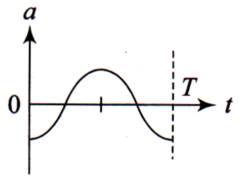
Applying conservation of energy

$$mgx - \frac{1}{2}kx^2 = 0 - 0 \Rightarrow x = \frac{2mg}{k}$$

20. **(c)** Displacement, $x = A\cos(\omega t)$ (given)

Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t)$

Acceleration, $a = \frac{dv}{dt} = -A\omega^2\cos(\omega t)$



Hence graph (c) correctly depicts the variation of a with t.

21. **(c)** The two displacement equations are $y_1 = a\sin(\omega t)$

and $y_2 = b\cos(\omega t) = b\sin\left(\omega t + \frac{\pi}{2}\right)$

$$y_{eq} = y_1 + y_2$$
$$= a\sin\omega t + b\cos\omega t$$
$$= a\sin\omega t + b\sin\left(\omega t + \frac{\pi}{2}\right)$$

Since the frequencies for both SHMs are same, resultant motion will be SHM.

Now $A_{eq} = \sqrt{a^2 + b^2 + 2ab\cos\frac{\pi}{2}}$

$$\Rightarrow \quad A_{eq} = \sqrt{a^2 + b^2}$$

22. **(a)** Maximum velocity $V_{max} = A\omega = \beta$ **(i)**

maximum acceleration $\alpha_{max} = A\omega^2 = \alpha$ **(ii)**

Equation (ii) divided by (i) $\omega = \frac{\omega}{\beta} \Rightarrow \frac{2\pi}{T} = \frac{\omega}{\beta}$

$$T = \frac{2\pi\beta}{\alpha}$$

23. **(a)** If initial length $l_1 = 100$ then $l_2 = 121$

By using $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$

Hence, $\frac{T_1}{T_2} = \sqrt{\frac{100}{121}} \Rightarrow T_2 = 1.1T_1$

$\% \text{ increase} = \frac{T_2 - T_1}{T_1} \times 100 = 10\%$

Alternative: Time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$$

$$\therefore \quad \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

Since, $\frac{\Delta l}{l} = 21\%$

$$\therefore \quad \frac{\Delta T}{T} = \frac{1}{2} \times 21\% \approx 10\%$$

24. **(d)** As springs are connected in series, effective force constant

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

25. **(c)** $n = \frac{1}{2\pi}\sqrt{\frac{K_{\text{effective}}}{m}}$

Springs are connected in parallel

$$K_{\text{eff}} = K_1 + K_2 = K + 2K = 3K$$

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{(K + 2K)}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$$

26. **(a)** As springs are connected in series, effective force constant

$$\frac{1}{k_{\text{eff}}} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \Rightarrow k_{\text{eff}} = \frac{k}{2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2M}}$$

27. For a simple harmonic motion

$$\frac{d^2y}{dt^2} \propto -y$$

Hence, equation $y = \sin\omega t - \cos\omega t$

and $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$ are satisfying this condition and equation $y = 1 + \omega t + \omega^2 t^2$ is not periodic and $y = \sin^3 \omega t$ is periodic but not SHM.

28. The motion of planets around the sun is periodic but not simple harmonic motion.

29. For freely falling case the effective g is zero, so that frequency of oscillation will be zero.

As $f = \frac{1}{2\pi}\sqrt{\frac{g_{\text{eff}}}{\lambda}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{0}{\lambda}}$$

$$f = 0$$

30. $x(t) = A\cos(\omega t + \phi)$

where, ϕ is the phase constant.

31. $y_1 = 5[\sin 2\pi t + \sqrt{3}\cos 2\pi t]$

$$= 10\left[\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right]$$

$$= 10\left[\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t\right]$$

$$= 10\left[\sin\left(2\pi t + \frac{\pi}{2}\right)\right] \Rightarrow A_1 = 10$$

Similarly, $y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

$\Rightarrow$ $A_2 = 5$

Hence, $\frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$

32. The potential energy, $U = \frac{1}{2}kx^2$

$$2U = kx^2$$

$$2U = -Fx \qquad (\because F = -kx)$$

or $\frac{2U}{F} = -x$

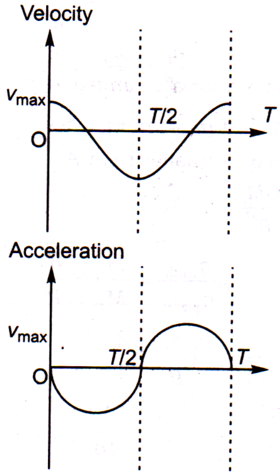
or $\frac{2U}{F} + x = 0$

33. Phase difference $\Delta\phi = \phi_1 - \phi_2$

$$= \frac{3\pi}{6} - \frac{\pi}{6}$$

$$= \frac{2\pi}{6} = \frac{\pi}{3}$$

34. In SHM, the acceleration is ahead of velocity by a phase angle $\frac{\pi}{2}$.



$$v = A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$a = A\omega^2 \sin(\omega t + \pi)$$

35. The total energy a particle executing SHM

$$= \frac{1}{2} m\omega^2 A^2$$

The PE of the particle at a distance x from the equilibrium position

$$= \frac{1}{2} m\omega^2 x^2$$

From the question, $\frac{1}{2} m\omega^2 x^2 = \frac{1}{2}\left(\frac{1}{2} m\omega^2 A^2\right)$

$\Rightarrow$ $$x^2 = \frac{A^2}{2} \Rightarrow x = \frac{A}{\sqrt{2}}$$

36. The average acceleration of a particle performing SHM over one complete oscillation is zero.

37. Let x be the point where KE = PE

Hence $$\frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2} m\omega^2 x^2$$

$$2x^2 = a^2,\ x = \frac{a}{\sqrt{2}}$$

$$x = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ cm}$$

38. By using $k \propto \frac{1}{l}$

Since, one-fourth length is cut away so remaining length is $\frac{3}{4}$th, hence k becomes $\frac{4}{3}$ times *ie*, $k' = \frac{4}{3} k$.

39. Maximum velocity $v_{max} = A\omega$

$$\omega = \frac{2\pi}{T}$$

$\therefore$ $$v_{max} = \frac{2\pi A}{T}$$

$$v \propto \frac{A}{T}$$

$\therefore$ $$\frac{v_1}{v_2} = \frac{A_1}{A_2} \times \frac{T_2}{T_1} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$v_2 = 6v_1 = 6v$$

40. For the given figure,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} \quad \ldots(i)$$

$$= \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

If one spring is removed, then $k_{eq} = k$ and

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{f}{f'} = \sqrt{2}$$

$$f' = \frac{1}{\sqrt{2}} f$$

41. $\frac{d^2x}{dt^2} + 16x = 0$

$\therefore$ $$\omega^2 = 16 \Rightarrow \omega = 4$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$$

42. In a complete cycle of SHM, potential energy varies for half the cycle and kinetic energy varies for the other half of the cycle. Thus, for a time period T, the potential energy varies for $\frac{T}{2}$ time.

43.
44.
45.

[CHEMISTRY]

46. The corresponding acids are HI, HCl, HNO_2 and HCN. Their acid strength follows the order HI > HCl > HNO_2 > HCN. Hence, their conjugate base follows the reverse order.

47.

pH of a weak acid is given by

(I) $2\text{pH} = \frac{1}{2}[pK_a - \log C]$ at $C = 0.1\text{M}$

(II) $\text{pH} = \frac{1}{2}[pK_a - \log C']$ at $C' = ?$

$\therefore$ $4\text{pH} = pK_a - \log C'$
$2\text{pH} = pK_a = \log C$

$2\text{pH} = \log C - \log C' = \log\frac{0.1}{C'}$

From I, $\text{pH} = \frac{1}{2}[4.74 - \log 0.1] = \frac{1}{2}[4.74 + 1.0] = 2.87$

$\therefore$ $2 \times 2.87 = \log\frac{0.1}{C'} \Rightarrow 5.74 = \log\frac{0.1}{C'}$

$\therefore$ $\frac{0.1}{C'} = 5.5 \times 10^5$

Thus, dilution $\frac{1}{C'} = \frac{5.55\times10^5}{0.1} = 5.55\times10^6$ times

48.

The conjugate acid-base pairs are (HCl, Cl^-) and ($CH_3COOH_2^+$, CH_3COOH).

49.

The conjugate acids are H_2O, NH_3, $HC \equiv CH$ and CH_3CH_3. Their order of acid strength is $CH_3CH_3 < NH_3 < HC \equiv CH < H_2O$. Their conjugate base follows the reverse order.

50.

NH_3 donates pair of electrons while BF_3, Cu^{2+} and $AlCl_3$ accept lone pair of electrons.

51.

Acid $\xrightarrow{-H^+}$ Conjugate base, Base $\xrightarrow{+H^+}$ Conjugate acid

52.

H_3O^+ (acid), H_2O (conjugate base) and not OH^-.

53.

54.

For each weak polyprotic acid $K_{a_1} > K_{a_2} > K_{a_3}$

55.

H_2O is the weaker base, hence, its conjugate acid is the stronger acid,

$H_2O + H^+ \rightleftharpoons H_3O^+$

H_2O is the weakest acid, hence, its conjugate base is the strongest base.

$H_2O \rightleftharpoons OH^- + H^+$

56.

pH [HCl] = 2.0

$\therefore$ $[H^+] = 10^{-2}$ M

$[HCl] = 10^{-2}$ M

Volume = 200 mL

pH [NaOH] = 12.0

pOH = 2.0

$[OH^-] = 10^{-2}$ M

$[NaOH] = 10^{-2}$ M

Volume = 300 mL

N_1V_1 (acid) = $200 \times 10^{-2} = 2$

N_1V_2 (base) = $300 \times 10^{-2} = 3$

$N_2V_2 > N_1V_1$

Thus, resultant mixture basic.

$$N(OH^-) = \frac{N_2V_2 - N_1V_1}{V_1 + V_2} = \frac{3-2}{500} = 2 \times 10^{-3}\ M$$

pOH = $-\log(2 \times 10^{-3}) = 2.7$

$\therefore$ pH = 14 – pOH 14 – 2.7 = 11.3

57.

$$[H^+] \text{ after mixing} = \frac{10^{-2} \times 10 + 10^{-4} \times 990}{1000} = \frac{0.1 + 0.0990}{1000}$$

$$= \frac{0.1990}{1000} = 1.99 \times 10^{-4}$$

pH = $(\log 1.99 \times 10^{-4})$

$\therefore$ pH = 4 – 0.3 = 3.7

58.

$$[H^+] = \frac{50 \times 10^{-1} + 50 \times 10^{-2}}{100} = 5.5 \times 10^{-2} M$$

pH = $\log(1.99 \times 10^{-4})$

$\therefore$ pH = 2 – 0.74 = 1.26

59.

On heating pure water the value of ionic product of water increases i.e., $K_w = 10^{-14}$ at 25°C and at 100°C, $K_w = 10^{-12}$. Thus pH and pOH both become 6 at 100°C (pH and pOH = 7 at 25°C).

60.

(a) At 25°C, $[H^+]$ in a solution of 10^{-8} M HCl > 10^{-7} M.

(b) $[H^+] = 10^{-8}$ M.

(c) $[OH^-] = 4 \times 10^{-6}$ M $\Rightarrow$ $[H^+] = 2.5 \times 10^{-9}$ M

(d) $[H^+] = 10^{-9}$ M

61.

K_w changes with temperature. As temperature increases, $[OH^-]$ and $[H^+]$ decrease.

62.

Meq. of HCl = $10 \times 10^{-1} = 1$

Meq. of NaOH = $10 \times 10^{-1} = 1$

Thus both are neutralised and 1 Meq. of NaCl (a salt of strong acid and strong base) which does not hydrolyse and thus pH = 7.

63.

The dissociation of H_3BO_3 is

$$H_3BO_3 + H_2O \rightarrow H_2BO_3^- + H_3O^+$$

$$K_1 = \frac{[H_2BO_3^-][H_3O^+]}{[H_3BO_3]} = \frac{(0.18).x}{(01.0)} = 7.3 \times 10^{-10}$$

or $x = [H_3O^+] = 4.1 \times 10^{-10}$

or $pH = -\log x = -\log(4.1 \times 10^{-10}) = 9.39$

64.

(a) HCl — NaOH

No. of milli eq. $= \frac{1}{10} \times 100 = 10$ — $\frac{1}{10} \times 100 = 10$

So solution is neutral

(b) $\frac{1}{10} \times 55 = 5.5$ — $\frac{1}{10} \times 45 = 4.5$

$[H^+] = \frac{1}{100} = 10^{-2}M,\ pH = 2$

(c) $\frac{1}{10} \times 10 = 1$ — $\frac{1}{10} \times 90 = 9$ Basic

(d) $\frac{1}{5} \times 75 = 15$ — $\frac{1}{5} \times 25 = 5$

$[H^+] = 0.1$ M, pH = 1

65.

Initial	Final
pH = 12	pH = 11
$[H^+] = 10^{-12}$ M	$[H^+] = 10^{-11}$ M
$[OH^-] = 10^{-2}$ M	$[OH^-] = 10^{-3}$ M

Initial no. of mole of $OH^- = 10^{-2}$

Final no. of mole of $OH^- = 10^{-3}$

So no. of mole of OH^- removed = [0.1 – 0.001] = 0.009

66.

$$pK_w = -\log K_w = -\log 1 \times 10^{-12} = 12.$$

$$K_w = [H^+][OH^-] = 10^{-12}$$

$$[H^+] = [OH^-]$$

$\Rightarrow$ $[H^+]^2 = 10^{-12}$; $[H^+] = 10^{-6}$; $pH = -\log[H^+] = -\log 10^{-6} = 6.$

H_2O is neutral because $[H^+] = [OH^-]$ at 373 K even when pH = 6.

(d) is not correct at 373 K. Water cannot become acidic.

67.

Relative strength of weak acids $= \sqrt{\left(\frac{K_{a_1}}{K_{a_2}} \times \frac{C_1}{C_2}\right)}$

$\therefore$ Relative strength $= \sqrt{\left(\frac{K_{a_1}}{K_{a_2}}\right)}\ (\because C_1 = C_2) = \sqrt{\left(\frac{2\times10^{-4}}{2\times10^{-5}}\right)}$

Relative strength for HCOOH to $CH_3COOH = \sqrt{10}:1$

68.

$$pH = 13$$

$\therefore$ $[H^+] = 10^{-13}$ M

$[OH^-] = 10^{-1}$ M = 0.1 mol L^{-1}

$[Ba(OH)_2] = 0.1$ N,

= 0.1 × 100 = 10 milliequivalents

69.

pH of amphiprotic salts and weak acid-weak base salt is independent of its concentration.

70.

71.

72.

Reaction: $2A + B \rightleftharpoons C + D$

$$K_P = \frac{n_C \times n_D}{n_A^2 \times n_B} \times \left(\frac{P}{\Sigma n}\right)^{\Delta n_g}$$

$$\Delta n_g = 2 - 3 = -1$$

$$K_P = \frac{n_C \times n_D}{n_A \times n_B} \times \left(\frac{\Sigma n}{P}\right)$$

$$PV = \Sigma nRT$$

$$\frac{V}{RT} = \frac{\Sigma n}{P}$$

From equations (i) and (ii),

$$K_p = \frac{n_C \times n_D}{n_A \times n_B} \times \frac{V}{RT}$$

73.

Concentration of $[NO_2]$ will decrease with increase in concentration $[N_2O_4]$.

74.

With passage of time conc. of reactants decreases and products increases.

75.

$$K = 2 = \sqrt{k_1}\ ,\ K_2 = \frac{1}{K_4}\ ,\ K_1 = \frac{1}{K_3}$$

$$\therefore \quad K_1K_3 = 1,\ \sqrt{K_1}\ K_4 = 1\ \sqrt{K_3}\ = 1$$

76.

$$\Delta n_g = 4 + 1 - (2 + 2) = 1$$

$$\therefore \quad K_P = k_c\,(RT)^{\Delta n_g}$$

$$0.03 = K_C\,(0.082 \times 700)^1$$

$$K_C = 5.23 \times 10^{-4}$$

77.

Required equilibrium is obtained if we operate.

Eq. (III) × 4 – Eq. (I) × 2 – Eq. (II) × 2

$$K_C = \frac{[N_2O_4]^2}{[N_2O]^2[O_2]^3} = \frac{(4.1\times10^{-9})^4}{(2.7\times10^{-18})^2\,(4.6\times10^{-3})^2} = 1.832\times10^6$$

78.

At equilibrium rates of backward and forward reactions become equal.

87. (c) HCl is a strong electrolyte since it will produce more H^+, comparison than that of CH_3COOH. Hence assertion is true but reason false.

88. (a) Barium carbonate is more soluble in HNO_3 than in water become carbonate is a weak base and reacts with the H^+ ion of HNO_3 causing the barium salt to dissociate.

$BaCO_3 + HNO_3 \rightarrow Ba(NO_3)_2 + CO_2 + H_2O$

89. (a) The conjugate base of $CHCl_3$ is more stable than conjugate base of $CHF_3(CF_3)$. CCl_3 stabilized by –I effect of chlorine atoms as well as by the electrons. But conjugate base of $CH_3(CH_3)$ is stabilized only by –I effect of flourine atoms. Here both assertion and reason are true and reason is correct explanation of assertion.